Analysis of contract negotiation of BOT projects

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Abstract

In general, the infrastructure projects are with high economic and social benefits, but with low financial viability. No need to say, the government needs to provide some incentives, such as subsidy or quarantine to allure the participation of private sectors on the infrastructure projects. Win-win situation is the best results for both parties. However, there is no attraction to private sectors if the government propose low subsidy, and it increase the government financial burden with high government subsidy.

There are three major participants in PPP projects, which are the government (the client), the Project Company, and bankers with different interests. We may say that it is a tri-party game in a PPP project. Thus, a cooperative game model is constructed to determine the government subsidy, tariff, debt ratio, and interest rate, which may lead to a win-win-win results.

A cable car project at Kaohsiung first harbor is used as a case study for constructing the financial model. The results show that the government subsidy =64%, tariff = 16.69 NT$, debt ratio = 46%, and interest rate = 8% in the case study. The project value is 1.73 billion NT$. The Shapley value is 1.67 billion NT$ for the government, 3.8 million NT$ for the project company, and 1.74 million for bankers. We may find a solution for this cooperative game, which implies that all parties meet their benefit requirements.

Keywords: Public private partnership (PPP), Game theory, Cooperative game, Shapley value, Project negotiation.

Introduction

According to Myerson (1991), game theory is the study of mathematical models of conflict and cooperation among rational and intelligent decision-makers. Each participant in the negotiations of PPI contracts tends to maximize his/her benefits, and the agreement can be reached only when satisfying all other participants' benefits. This is a Nash equilibrium, which indicates a player does his/her best, given that his/her rivals also do their best (Narahari, 2007). However, the negotiating parties might not reach agreements (no Nash equilibrium) or equilibria (multiple Nash equilibria).

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In the analysis of games with more than two players, von Neumann (1959) assumed that players would not simply choose their strategies independently, but they would coordinate their strategies in coalitions. Like traditional game theory, the approach advocated here builds on the notions of individual preferences and individual rationality. Those who participate in the PPI scheme formulate the project together, realizing that they have the project in common and that they depend on each other. This rationality of the participants is depicted in two ways. First, participants prepare a proposal about how they would formulate the project and which conjectures they would follow regarding each other's behavior. Second, if a proposal were unambiguously in everyone's interest, then the individual participants would not hesitate to choose their part in the proposal. What is in a participant's own interest is simply defined by its own utility function in the PPI project, and participants only consider proposals that are consistent with individual rationality.

Let $N = \{1,2,3,..., n\}$ be the set of PPI participants and F be the set of feasible payoffs that the participants can receive if they all work together. Let $(v_1, v_2, v_3,..., v_n)$ be the disagreement payoffs the participants would expect if they did not cooperate. In addition, assume that the set $\{(x_1, x_2, x_3,..., x_n) \in \mathbf{F} : x_i \geq v_i, \forall i \in N\}$ is non–empty and bounded. The pair $\mathbf{F}, (v_1, v_2, v_3,..., v_n)$ is n–person bargaining problem (Narahari, 2007). The Nash bargaining equilibrium can be defined as an efficient allocation vector that maximizes

$$\sum_{i}^{n} (x_i - v_i)$$

over all factors $x \in \mathbf{F}$ such that $x_i \geq x_i^{\min}, \forall i \in N$.

Next, we incorporate the cooperation among the participants into the above solutions. Assume that the participants wish to divide the project benefits among themselves. Each participant can propose a payoff such that no participant's payoff is lower than its required minimum ($x_i^{\min}$). Let the social NPV (SNPV) present the total payoffs allocated among n parties:

$$SNPV = FNPV + [(TAX - SUB) + UB] + IE$$

where SNPV denotes social benefits; (TAX – SUB)+UB are public benefits governed by the government; FNPV are the benefits going to the sponsor; and IE are the earnings of the financial institutions.

The objective of Equation (5) is to maximize SNPV. The success of the project depends on the strategy ($\|\mathbf{S}\|$), which determines the major variables, including tariff (y1) of the services offered by the project, the interest rate (y2), the debt ratio (y3), and the subsidies (y4). The strategy sets can be defined as follows:

$$\mathbf{S}_1 = \mathbf{S}_2 = \mathbf{S}_3 = ... = \mathbf{S}_n = \{(y_1, y_2, y_3, y_4) \in R^4 : x_1 + x_2 + x_3 + ... + x_n \leq SNPV^{Max}$$

$$x_1 \geq x_1^{\min}; x_2 \geq x_2^{\min}; x_3 \geq x_3^{\min};...; x_n \geq x_n^{\min}\},$$

where S1, S2,S3,…,and Sn denote the strategy sets for each of the n parties, respectively, and x1, x2, x3 , … and xn present the corresponding payoffs.

Assume that the participants will receive 0 payoffs, unless they propose the same solution (i.e., the strategy). That is, for $i = (1,2,3,..., n)$, the outcome of the game F($\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$, … , $\mathbf{S}_n$) will be

$$F_i(\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3,..., \mathbf{S}_n) = \begin{cases} x_i & \text{if } \mathbf{S}_1 = \mathbf{S}_2 = \mathbf{S}_3 = ... = \mathbf{S}_n = (y_1, y_2, y_3, y_4) \\ 0 & otherwise \end{cases}$$

In this model, the participants may achieve a solution in which each of their payoffs is greater than a minimum value and the total payoffs are maximized. The minimax payoff guaranteed for each participant is greater than 0. The equation (7) can be described further as n–party bargaining problem with payoff function (F), with $v_i, (i = 1,2,3,..., n)$ denoting disagreement payoffs of n parties, as follows

$$F = \{(x_1, x_2, x_3,..., x_n) \in R^n : x_1 + x_2 + x_3 + ... + x_n \leq SNPV^{Max},$$

$$x_1 \geq x_1^{\min}; x_2 \geq x_2^{\min}; x_3 \geq x_3^{\min};...; x_n \geq x_n^{\min}\}$$

$$v = (v_1, v_2, v_3,..., v_n) = (0,0,0,...,0)$$

The n–party game is a simultaneous move game; however, not all participants have the same bargaining powers. The participants with high bargaining power are dominant players while those with low bargaining power are the followers. A game is similar to a triopoly game, where three participants in the n–party game are dominant players while a duopoly is a game that requires only two dominant players. A monopoly game has only one dominant player, that is, it can be considered a dictator game.

- The core and Shapley value for cooperative game

In cooperative game theory, we abstract from individual players' strategies and instead focus on the coalition players may form. We assume each coalition may attain some payoffs, and then we try to predict which coalitions will form and hence the payoffs agents obtain. Practically speaking, strategic game theory deals with various equilibrium concepts and is based on a precise description of the game in question. Coalitional game theory deals with concept like the core, Shapley value, the Nash-bargaining solution.

Core is a solution concept that assigns to each cooperative game the set of payoffs that no coalition can improve upon or block. In a context in which there is unfettered coalitional interaction, the core arises as a good positive answer to the question posed in cooperative game theory. In other words, if a payoff does not belong to the core, one should not expect to see it as the prediction of the theory. The core of a game is defined as

$$\text{core}(\upsilon) \equiv \left\{(x_1, \dots, x_n)/\sum_{i=1}^{n} x_i \leq v(N), \forall S, \sum_{i\in S} x_i \geq v(S)\right\}$$

The core is an allocation (x1,…,xn), where xi is the payoff for player i, of total surplus υ(N), that satisfies:

It is feasible, $\sum_{i=1}^{n} x_i \leq v(N)$

A set of players S obtain at least what they would obtain forming a coalition S, $\forall S, \sum_{i\in S} x_i \geq v(S)$. Otherwise, they will not accept the allocation and would blockade its formation.

Sometimes the core does not give a unique prediction. Sometimes, the core is empty. In a 3-party game, the core is shown in equation (3.40).

$$v(\mathrm{A}) \leq X_{\mathrm{A}} \leq v(ABC) - v(N\backslash\{\mathrm{BC}\})$$

$$v(\mathrm{B}) \leq X_{\mathrm{B}} \leq v(ABC) - v(N\backslash\{\mathrm{AC}\})$$

$$v(\mathrm{C}) \leq X_{\mathrm{C}} \leq v(ABC) - v(N\backslash\{\mathrm{AB}\})$$

Shapley value is a solution that prescribes a single payoff for each player, which is the average of all marginal contributions of that player to each coalition he or she is a member of. It is usually viewed as a good normative answer to the question posed in cooperative game theory. That is, those who contribute more to the groups that include them should be paid more.

Shapley value is a function '(v) which assigns each player a number/value indicating the relative power of that player in the game as average marginal power. In general, the Shapley value of a game with n players is defined as:

$$\varphi_i = \sum_{i\in S\subseteq N} \frac{(n-|S|)!\,(|S|-1)!}{n!}\big(v(S) - v(S-\{i\})\big)$$

Where n is the number of players, and |S| is the number of players in set S, υ(s) is the payoffs in set S.

- The bargaining solutions for 2-party game

Assume the set of feasible payoffs is denoted by u, which we will assume to be convex, the threat points of the two players by $\mathbf{v} = (v_1, v_2)$, and the Nash bargaining solution by $\mathbf{u}^* = (u_1^*, u_2^*)$. This latter is shown to be a function of $\mathbf{v}$ and u, and follows from axioms that Nash regards as 'fair' and reasonable conditions to be fulfilled by rational parties. The basic axioms are individual rationality, strong efficiency, symmetry, scale invariance, and independence of irrelevant alternatives. Nash showed that there is only one possible solution that satisfies all these conditions, this is the Nash point, N. Assuming both parties have zero threat points (i.e. $v_1 = v_2 = 0$) then the Nash bargain solves the problem:

$$\arg\max F = u_1 u_2$$

$$\text{s.t. } u_i \in u, u_i \geq v_i$$

More generally, assume that the parties have positive threat points (i.e. $v_1, \ v_2 > 0$) then the Nash bargain solves the problem:

$$\arg\max F = (u_1 - v_1)(u_2 - v_2)$$

s.t.

$$u_i \in u, u_i \geq v_i$$

The objective function in (3.43), referred to as the Nash-Product, is continuous and strictly quasi-concave. The feasible set in (3.43) is non-empty, closed, bounded and convex. It follows then that the solution to the optimization program exists and is unique.

For 2-party game by following the axiomatic model of Nash, we take the pair $(\mathbf{u}, \mathbf{v})$) to define a bargaining problem, and assume that u is compact and convex. The generalized Nash solution (Osborne and Rubinstein, 1990) is given by

$$\psi_G(F, v) = \underset{u \in F, u \geq v}{\arg\max} (u_1 - v_1)^{\alpha} (u_2 - v_2)^{\beta} \; for \; G = 1,2$$

Where α and β denotes the bargaining power of each participants in project negotiation. And, β+α=1. The strategy can be determined by the first order condition.

$$\alpha[u_2(\gamma) - v_2] u_1'(\gamma) + \beta[u_1(\gamma) - v_1] u_2'(\gamma) = 0$$

Empirical Study

To simplify the computation, we assumed a build–own–operate (BOO) project. This means that a private sponsor builds the facility and then owns and operates it infinitely. We collected the data from the pre–feasibility study (Bureau of Metropolitan Development of Kaoshung City Government, 2004) of the Kaohsiung International Intelligence Free Port Project (hereinafter we call “the Project”) in Taiwan and applied it to the cooperative game model. This project has been proposed to ease the heavy traffic flow on the cross–harbor tunnel, linking Cijin and Kaohsiung City. Cijin, located southwest of Kaohsiung City across the sea, is a tourism destination in southern Taiwan known particularly for its seafood. Currently, the only way to move between Cijin and Kaohsiung City is by sea–faring vessels through the cross–harbor tunnel where the traffic is very heavy, especially on the weekends. The Project aimed to build an inter–port tramway, utilizing sightseeing cable cars between Cijin and Kaohsiung City to help offset the congestion on the seafaring vessels.

The solutions for the tropoly game model are presented in Tables 2 and 3. Table 2 shows the payoffs of the three parties at different levels of government subsidies while Table 3 gives the solutions for the decision variables. Figure 2 shows that SW of the project increased together with the government subsidy before it reached a maximum value when SUB ratio is 66.55%. It decreased thereafter. In addition, tariff of the ridership in the project decreased as the government subsidy increased. At the optimal solution, the project capacity was fully used, the tariff was NT$76.27, debt ratio was 30.84%, and interest rate was 6.35%. The total social benefits under optimal government subsidies were 8.5 times greater compared to those of the no–subsidy case. These results imply that optimal government subsidy of the project exists. This is because the capacity of the project was used fully at the optimal solution. No passenger benefits can be created by additional government subsidies.

Table 1: Benefits of three parties with various government subsidy for the three-party game model

SUB	SB	TAX	FNPV	TB	INT
0	744.83	1305.27	0	2161.10	111.00
0.1	4171.73	1164.72	0	5335.72	100.47
0.2	7053.97	1024.23	0	7965.37	89.57
0.3	9693.72	883.75	0	10352.40	78.53
0.4	12187.44	743.29	0	12593.34	67.41
0.5	14580.77	602.83	0	14733.84	56.24
0.6	16899.60	462.37	0	16799.80	45.03
0.7	17472.20	422.09	46.72	17304.30	71.69
0.8	17472.20	407.75	154.02	17254.37	33.04
0.9	17472.20	432.02	245.26	17251.98	13.30

1.0	17472.20	434.28	335.93	17230.41	0.00

The solutions for the duopoly game model are presented in Tables 4 and 5. At the optimal solution, the tariff was NT$76.26, debt ratio was 30.40%, and interest rate is 6.30%. The results showed that solutions for these two models are almost the same (Table 6). The results demonstrated that when negotiating PPI contracts, financial institutions have smaller bargaining power compared to the government and private investors. In conclusion, duopoly game can be used to find the optimal solutions for PPIs.

The total benefit equals to 17,265 Million NT$ in optimization case and 2,051 Million NT$ in base case of the two–party model. The total benefit increased about 8.5 times in value, proving that game models are useful in discovering the optimal value of the PPI projects. Similar results were found for three–party game model. An interesting finding is that debt ratio turned out to be 27.87% in two–party game model and 46.63% in three–party game model, which is almost 2–fold. The interest rate also increased from 6.03% in two–party game model to 8.04% in three–party model.

7.4. Discussions and Conclusion

While game model can be used to find the optimal solutions for negotiating PPI contracts, the type of the game is subject to the bargaining power among the participants. For the cable car project, we found that the public benefit has a dominant influence on the solutions of the game model whereas tariff plays an important role in determining the maximum public benefits. With the assumption of negative price elasticity of the demand for cable car, the number of passengers increases with lower tariff. When the capacity of the cable car facilities is fully utilized, the marginal benefits driven by government subsidies drop to zero and the total social welfare diminishes thereafter. Thus, total benefit of the PPI project reaches a maximum value. We proposed a game model that includes the benefits of the three major PPI parties. The cable car project shows that total social benefits are greater under optimal government subsidies compared to those without subsidy. This is because government subsidies lower the tariff and increase ridership. The model focuses on the financial factors, including the government subsidy, tariff, debt ratio, and interest rate for PPI infrastructure projects.

Our model provides an effective approach to find optimal solutions and determine the major financial variables. Our research had certain limitations. We dropped financial institutes from the game model because their stakes in the total social benefit are small. In the duopoly game model, we did not give weights to the bargaining power of the two sides of the game. In addition, we did not consider political issues that may affect concession variables, such as tariff, the ceilings of which are more politically sensitive compared to general financial variables. Finally, we did not incorporate project risk into the model. Releasing these constraints would result in a more complicated game model and complex calculation.

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